Exam Preparation: Circular Motion, Gravitation, Optics

Theory: Know these concepts by heart

a) Vector/scalar
b) Explain the meaning of "inertia" as a property of mass.
c) Explain the meaning of "gravity" as a property of mass.
d) Definition of work
e) Light ray
f) Give examples of luminous and illuminated objects
g) How do we see an object?
h) Magnification
i) Period
j) Frequency
k) Angular velocity
l) Centripetal acceleration
m) Centripetal force
n) Gravitation
o) Which force provides the centripel force that keeps a satellite in orbit around the Earth?

Physical quantities: Know these physical quantities by heart (symbol and unit)

	symbol	unit		symbol	unit
time			distance, displacement		
velocity			acceleration		
acceleration of free fall			force		
mass			density		
work			energy		
power			distance, radius		
period			frequency		
angle in radian measure			angular velocity		
centripetal acceleration			centripetal force		
gravitational force			magnification		
image height			object height		
image distance			object distance		

Formulae: A formula sheet will be handed out. Please find the formula sheet on massenpunkt.ch.

Skills:

- transform equations, insert numbers with units into the equation, calculate results correctly
- round your results to the correct number of significant digits and write your answer with a power of ten in the normalized scientific format
- draw and read scientific graphs
- represent vectors graphically by drawing them as arrows and solve problems by using this method
- Draw a free-body-diagram to show all the forces acting on an object (representing the forces as arrows)
- Determine the resultant of several vectors, as well as the components of a vector, using their graphical representation as arrows
- construct image formation by plane mirrors
- construct image formation by a pinhole camera

Exercises:

An algebraic solution and all values used in calculations are required to get the full mark.

All work sheets, and assignment sheets A22 – A24

Additional problems

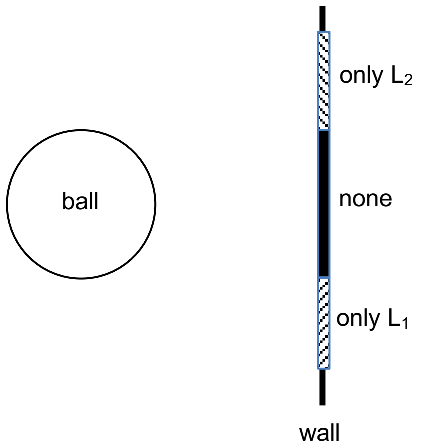
1. A tree of 1.7600 m height is located at a distance of 0.00530 km in front of the opening of a pinhole camera. The image distance is 0.0180650 m. The image height is to be calculated.

a) Place a dot above the digits which are significant. How many significant digits do the values which are needed in the calculation have? How many significant figures does your final answer require?

b) Calculate the image height (in meters) and round your result to the correct number of significant figures.

c) Write your result in the normalized scientific notation (with a power of ten).

2. A ball that is illuminated by two light sources L_1 and L_2, casts a shadow on the wall. Indicated in the picture are the places from where none of the light sources, or just one of them, respectively, can be seen.
Draw the positions of the two light sources and label them L_1 und L_2.



3. A tree of 6.0 m height is photographed with a pinhole camera. The screen where the image is formed is 4.3 cm behind the opening of the camera, and the image's height is 24 mm.

a) Calculate the magnification.

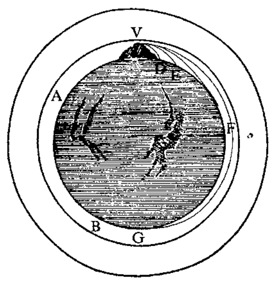
b) What is the distance between the opening of the pinhole camera and the tree?

4. Draw to scale: A figure of 4.0 cm height is placed in front of a pinhole camera. The figure and the back side of the camera (where the image is formed) are 14.0 cm apart. The magification is 0.75.
Determine d_i, d_o and h_i.

5. Mr. Hillbilly who is 1.80 m tall is standing between a wall and a light source. His shadow on the wall is 2.10 m high. Mr. Hillbilly and the wall are 45 cm apart.
What is the distance between Mr. Hillbilly and the lamp?

6. You've always wanted to know the mass of the person sitting next to you in school … If you (m = 65.0 kg) were sitting at 60.0 cm away from him, you'd "feel" (if you could!) an attractive force of 1.20 µN…
What is your schoolmate's mass?

7. Isaac Newton imagined shooting a cannon ball parallel to the earth's surface from the top of a high mountain. His hypothesis was that if the cannon ball's speed was high enough it would make its way around the earth without falling to the ground. It would stay in orbit just like the moon.
What is the speed needed for the cannon ball to move around the earth in a circular orbit? (ignore air resistance)



8. "Apollo 11 was the first mission in which humans walked on the lunar surface and returned to Earth. On 20 July 1969 two astronauts (Apollo 11 Commander Neil A. Armstrong and LM pilot Edwin E. "Buzz" Aldrin Jr.) landed in Mare Tranquilitatis (the Sea of Tranquility) on the Moon in the Lunar Module (LM) while the Command and Service Module (CSM) (with CM pilot Michael Collins) continued in lunar orbit. During their stay on the Moon, the astronauts set up scientific experiments, took photographs, and collected lunar samples. The LM took off from the Moon on 21 July and the astronauts returned to Earth on 24 July."

from: https://nssdc.gsfc.nasa.gov/nmc/spacecraft/display.action?id=1969-059A
(August 30th 2021)

While the Lunar Module descended to the Moon, the Command and Service Module kept moving around the moon in a circular path, 110.5 km above the Moon's surface.
Calculate the orbital period of the CSM.

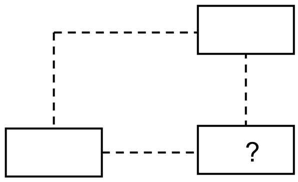
9. When travelling from Earth to the Moon the pull of gravity on the spacecraft first decreases, then disappears and then increases again.
a) Why?
b) How far away from Earth is this point where the net force on the spacecraft is zero?

10. An astronaut who was doing repair work on the outside of her spaceship has moved away from it by mistake and is now all alone in space, far away from everything else. The wrench has slipped from her hand and is now orbiting her with radius 1.02 m (just out of reach), once per day (T = 24.0 h).
Calculate the astronaut's mass.

11. Vesta ($m = 2.7 \cdot 10^{20}$ kg, $g = 0.23\ \frac{\text{m}}{\text{s}^2}$) is a planetoid that orbits the sun with a period of 1'326 d.
What is the diameter of Vesta?

12. The centers of two lead bullets are spaced 5.50 cm apart. The lead bullet on the left is three times heavier than the lead bullet on the right. They attract each other with a gravitational force of $3.97 \cdot 10^{-9}$ N.
What is the mass of the lead bullet on the right?

13.



difficult Three identical cars of mass m = 1.2 t each are parked on the corners of a rectangle with sides 1.25 m and 2.00 m (the fourth spot remains empty, see picture). What is the magnitude of the net gravitational force acting on the „?"-car?

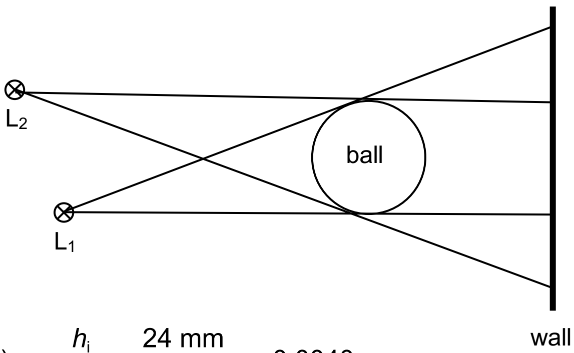
Solutions:

1. a) $h_o = \dot{1}.\dot{7}\dot{6}\dot{0}\dot{0}$ m (5 significant figures); $d_o = 0.00\dot{5}\dot{3}\dot{0}$ (3 significant figures); $d_i = 0.0\dot{1}\dot{8}\dot{0}\dot{6}\dot{5}\dot{0}$ (6 significant figures); result: 3 significant figures

   b) $h_i = \dfrac{h_o \cdot d_i}{d_o} = \dfrac{1.7600\text{ m} \cdot 0.0180650\text{ m}}{5.30\text{ m}} = 0.005998943\text{ m} = \underline{\underline{0.00600\text{ m}}}$

   c) $h_i = \underline{\underline{6.00 \cdot 10^{-3}\text{ m}}}$

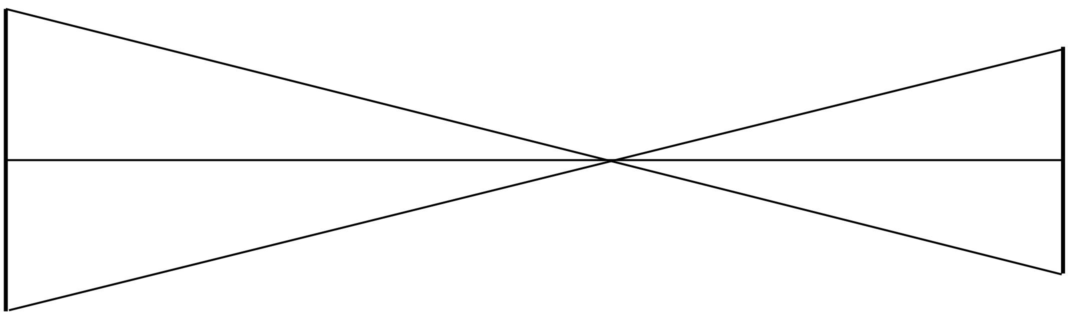
2.



3. a) $m = \dfrac{h_i}{h_o} = \dfrac{24\text{ mm}}{6'000\text{ mm}} = \underline{\underline{0.0040}}$

   b) $d_o = \dfrac{h_o \cdot d_i}{h_i} = \dfrac{6'000\text{ mm} \cdot 43\text{ mm}}{24\text{ mm}} = 10'750\text{ mm} = \underline{\underline{10.75\text{ m}}}$

4.



h_i = 3.0 cm, d_i = 6.0 cm, d_o = 8.0 cm

5. $m = \dfrac{h_i}{h_o} = \dfrac{d_i}{d_o} = \dfrac{210\text{ cm}}{180\text{ cm}} = \dfrac{7}{6}$

$d_i - d_o = \Delta d = 45\text{ cm}$ $\qquad$ $d_i = \Delta d + d_o$

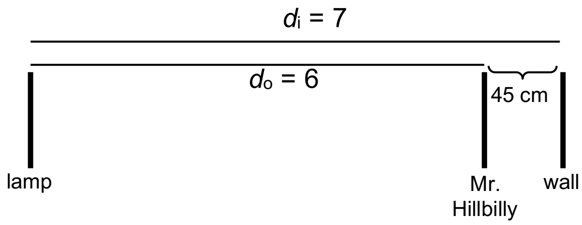
$\dfrac{h_i}{h_o} = \dfrac{d_o + \Delta d}{d_o}$ $\qquad$ $\dfrac{h_i}{h_o} \cdot d_o = d_o + \Delta d$ $\qquad$ $\dfrac{h_i}{h_o} \cdot d_o - d_o = \Delta d$ $\qquad$ $d_o \cdot \left(\dfrac{h_i}{h_o} - 1\right) = \Delta d$

$d_o \cdot \left(\dfrac{h_i - h_o}{h_o}\right) = \Delta d$ $\qquad$ $d_o = \dfrac{\Delta d \cdot h_o}{h_i - h_o} = \dfrac{45\text{ cm} \cdot 180\text{ cm}}{210\text{ cm} - 180\text{ cm}} = 270\text{ cm} = \underline{\underline{2.70\text{ m}}}$

or:

$m = \dfrac{h_i}{h_o} = \dfrac{d_i}{d_o} = \dfrac{210\text{ cm}}{180\text{ cm}} = \dfrac{7}{6}$

$d_o = 6 \cdot 45\text{ cm} = \underline{\underline{2.70\text{ m}}}$



6. $m_{\text{schoolmate}} = \dfrac{F_G \cdot r^2}{G \cdot m_{\text{you}}} = \dfrac{1.20 \cdot 10^{-6}\text{ N} \cdot (0.600\text{ m})^2}{6.67 \cdot 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot 65.0\text{ kg}} = \underline{\underline{99.6\text{ kg}}}$

7. If an object orbits the Earth, the centripetal force is provided by the gravitational force: $F_C = F_G$

$F_c = \dfrac{m_{\text{cannonball}} \cdot v^2}{r_{\text{Earth}}}$ $\qquad$ $F_G = G \cdot \dfrac{m_{\text{cannonball}} \cdot m_{\text{Earth}}}{r^2_{\text{Earth}}}$ $\qquad$ $\dfrac{m_{\text{cannonball}} \cdot v^2}{r_{\text{Earth}}} = G \cdot \dfrac{m_{\text{cannonball}} \cdot m_{\text{Earth}}}{r^2_{\text{Earth}}}$

$v = \sqrt{G \cdot \dfrac{m_{\text{Earth}}}{r_{\text{Earth}}}} = \sqrt{6.67 \cdot 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot \dfrac{5.97 \cdot 10^{24}\text{ kg}}{6.37 \cdot 10^{6}\text{ m}}} = 7'906\ \dfrac{\text{m}}{\text{s}}$

8. $r = r_{\text{Moon}} + h = 1'737 \cdot 10^3\text{ m} + 110.5 \cdot 10^3\text{ m} = 1.8475 \cdot 10^6\text{ m}$

$T = \sqrt{\dfrac{(2\pi)^2 \cdot r^3}{G \cdot m_{\text{Moon}}}} = \sqrt{\dfrac{(2\pi)^2 \cdot \left(1.8475 \cdot 10^6\text{ m}\right)^3}{6.67 \cdot 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot 7.346 \cdot 10^{22}\text{kg}}} = \underline{\underline{7'128\text{ s}}}$

9. a) The net force is zero where the attractive force of the moon equals the attractive force of the Earth.

b) $F_{G(\text{spacecraft-earth})} = F_{G(\text{spacecraft-moon})}$ $\qquad$ $r_{\text{moon-earth}} = r_{\text{spacecraft-earth}} + r_{\text{spacecraft-moon}}$

$F_{G(\text{spacecraft-earth})} = G \cdot \dfrac{m_{\text{spacecraft}} \cdot m_{\text{earth}}}{r^2_{\text{spacecraft-earth}}}$

$F_{G(\text{spacecraft-moon})} = G \cdot \dfrac{m_{\text{spacecraft}} \cdot m_{\text{moon}}}{r^2_{\text{spacecraft-moon}}} = G \cdot \dfrac{m_{\text{spacecraft}} \cdot m_{\text{moon}}}{\left(r_{\text{moon-earth}} - r_{\text{spacecraft-earth}}\right)^2}$

$G \cdot \dfrac{m_{\text{spacecraft}} \cdot m_{\text{earth}}}{r^2_{\text{spacecraft-earth}}} = G \cdot \dfrac{m_{\text{spacecraft}} \cdot m_{\text{moon}}}{\left(r_{\text{moon-earth}} - r_{\text{spacecraft-earth}}\right)^2}$

$\dfrac{r^2_{\text{spacecraft-earth}}}{\left(r_{\text{moon-earth}} - r_{\text{spacecraft-earth}}\right)^2} = \dfrac{m_{\text{earth}}}{m_{\text{moon}}}$ $\qquad$ $r_{\text{spacecraft-earth}} = \sqrt{\dfrac{m_{\text{earth}}}{m_{\text{moon}}}} \cdot \left(r_{\text{moon-earth}} - r_{\text{spacecraft-earth}}\right)$

$r_{\text{spacecraft-earth}} \cdot \left(1 + \sqrt{\dfrac{m_{\text{earth}}}{m_{\text{moon}}}}\right) = \sqrt{\dfrac{m_{\text{earth}}}{m_{\text{moon}}}} \cdot r_{\text{moon-earth}}$

$r_{\text{spacecraft-earth}} = \dfrac{\sqrt{\frac{m_{\text{earth}}}{m_{\text{moon}}}}}{1 + \sqrt{\frac{m_{\text{earth}}}{m_{\text{moon}}}}} \cdot r_{\text{moon-earth}} = \dfrac{\sqrt{\frac{5.972 \cdot 10^{24}\text{ kg}}{7.346 \cdot 10^{22}\text{ kg}}}}{1 + \sqrt{\frac{5.972 \cdot 10^{24}\text{ kg}}{7.346 \cdot 10^{22}\text{ kg}}}} \cdot 3.844 \cdot 10^8\text{ m} = \underline{\underline{3.460 \cdot 10^8\text{ m}}}$

10. $$m_{\text{Astronaut}} = \frac{4\pi^2 \cdot r^3}{T^2 \cdot G} = \frac{4\pi^2 \cdot (1.02\text{ m})^3}{(24 \cdot 3'600\text{ s})^2 \cdot 6.67 \cdot 10^{-11}\ \frac{\text{Nm}^2}{\text{kg}^2}} = \underline{\underline{84.1\text{ kg}}}$$

11. $$r_{\text{Vesta}} = \sqrt{G \cdot \frac{m_{\text{Vesta}}}{g}} = \sqrt{6.67 \cdot 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot \frac{2.7 \cdot 10^{20}\text{ kg}}{0.23\ \frac{\text{m}}{\text{s}^2}}} = 279'821\text{ m} = 280\text{ km}$$

$d = 2 \cdot r = 2 \cdot 279'821\text{ m} = 559'643\text{ m} = \underline{\underline{560\text{ km}}}$

12. $m_{\text{left}} = 3 \cdot m_{\text{right}}$ $\qquad F_G = G \cdot \frac{m_{\text{left}} \cdot m_{\text{right}}}{r^2} = G \cdot \frac{3 \cdot m_{\text{right}}^2}{r^2}$

$$m_{\text{right}} = \sqrt{\frac{F_G \cdot r^2}{3 \cdot G}} = \sqrt{\frac{3.97 \cdot 10^{-9}\text{ N} \cdot (0.055\text{ m})^2}{3 \cdot 6.67 \cdot 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}}} = \underline{\underline{0.245\text{ kg}}} = \underline{\underline{245\text{ g}}}$$

13. $$F_{G(2.00\text{m})} = G \cdot \frac{m^2}{r^2} = 6.67 \cdot 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot \frac{(1'200\text{ kg})^2}{(2.00\text{ m})^2} = 2.40 \cdot 10^{-5}\text{ N}$$

$$F_{G(1.25\text{m})} = G \cdot \frac{m^2}{r^2} = 6.67 \cdot 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot \frac{(1'200\text{ kg})^2}{(1.25\text{ m})^2} = 6.15 \cdot 10^{-5}\text{ N}$$

$$F_{G(\text{res})} = G \cdot m^2 \cdot \left(\sqrt{\frac{1}{r_1^4} + \frac{1}{r_2^4}}\right)$$

$$= 6.67 \cdot 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot (1'200\text{ kg})^2 \cdot \left(\sqrt{\frac{1}{(1.25\text{ m})^4} + \frac{1}{(2.00\text{ m})^4}}\right) = \underline{\underline{6.60 \cdot 10^{-5}\text{ N}}}$$