Exam Preparation: Simple machines, Circular motion

Theory: Know these concepts by heart

a) Vector/scalar
b) Explain the meaning of "inertia" as a property of mass.
c) Explain the meaning of "gravity" as a property of mass.
d) Definition of work
e) Power
f) Energy
g) Block and tackle
h) Lever
i) Give examples of levers from daily life
j) torque (moment of force)
k) lever arm
l) Give examples of a uniform circular motion
m) Period
n) Frequency
o) Angular speed
p) Centripetal acceleration
q) Centripetal force
r) What is the condition for a ball to continue in a circular path (and not fall down) in the upmost point, if it is whirled around on a string in a vertical circle? Or: what is the condition for a roller coaster to not fall down in the upmost point of a looping?

Physical quantities: Know these physical quantities by heart (symbol and unit)

	symbol	unit		symbol	unit
time			distance, displacement		
velocity			acceleration		
acceleration of free fall			mass		
force			gravitational force		
work			power		
energy			efficiency		
lever arm			torque (moment of force)		
angle in radian measure			angle in degree measure		
period			frequency		
angular speed			distance, radius		
centripetal acceleration			centripetal force		

Formulae: A formula sheet will be handed out. Please find the formula sheet on massenpunkt.ch.

Skills:

- Transform equations, insert numbers with units into the equation, calculate results correctly
- Round your results to the correct amount of significant digits and write your answer with a power of ten in the normalized scientific format
- Draw and read scientific graphs
- represent vectors graphically by drawing them as arrows and solve problems by using this method
- Draw a free-body-diagram to show all the forces acting on an object (representing the forces as arrows)
- Determine the resultant of several vectors, as well as the components of a vector, using their graphical representation as arrows
- Determine the correct number of weight-carrying ropes in a block and tackle

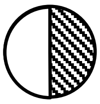
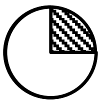
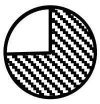
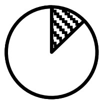
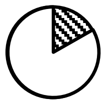
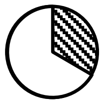
Exercises:

An algebraic solution and all values used in calculations are required to get the full mark.

All of the work sheets and assignment sheets A19 – A21

Additional problems

1. Give the angles shown in the picture (shaded parts) in radians as well as in degrees.

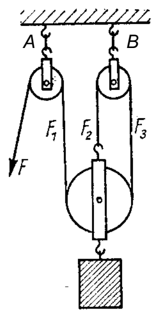
a)  b)  c)  d)  e)  f) 

2. Complete the following sentence:

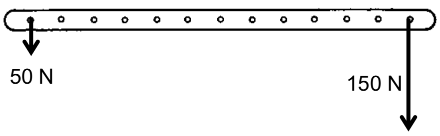
"In a uniform circular motion the .... of the velocity vector remains constant while its .... continuously changes. The acceleration is directed towards the ...."

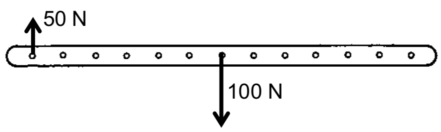
3. A box (m = 97.00 kg) is lifted through a height of 12.00 m, using a block and tackle. Each pulley has a mass of 550 g. Neglect friction.

a) Determine the force F that is applied to the rope.
b) What are the forces F_1, F_2, and F_3 acting on the rope?
c) Determine the forces acting on the suspensions A and B.
d) What is the length of the rope pulled downward at F?



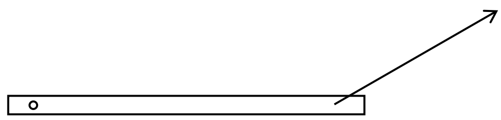
4. Determine the location of the axis of rotation (pivot) for the lever to be balanced.

a) 

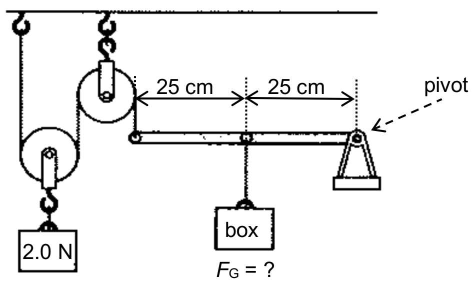
b) 

5. In the case that the applied force is not perpendicular to the lever arm, the lever arm can be substituted by two components: one component that is parallel to the line along which the force acts (exerting no torque) and one component that is perpendicular to the force. Measures: Lever arm: 1.0 cm corresponds to 1.0 m, force: 1.0 cm corresponds to 1.0 N

a) Substitute the lever arm graphically by two components: One that is parallel and one that is perpendicular to the line along which the force acts. Determine their magnitudes.
b) Calculate the moment of force (torque).
c) Substitute the arrow representing the force graphically by two components: One that is parallel and one that is perpendicular to the lever arm. Determine their magnitudes.
d) Calculate the moment of force (torque).
e) Compare b) and d).



6.



The system shown to the left is at equilibrium. Assume that the lever, rope and pulleys have no mass. What is the weight of the box?

7. A car accelerates from 0 to 102.97500 $\frac{\text{km}}{\text{h}}$. The work done by the motor is 0.4500 MJ. The car's mass is to be calculated.

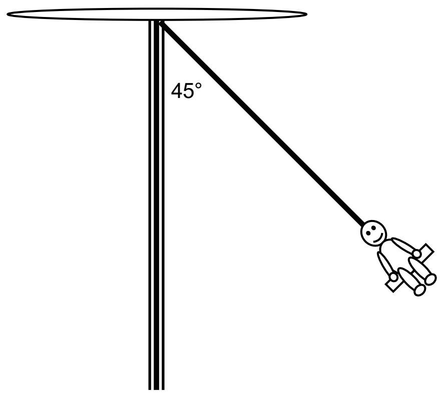
a) Place a dot above the significant figures of the values which are required in the calculation. How many significant digits do they have? How many significant figures does your final answer require?
b) Calculate the car's mass in kg.
c) Round your result to the correct number of significant figures and write it in the normalized scientific notation (with a power of ten).

8. Someone swings a little stone (m = 233 g) on a string of 1.79 m length in a horizontal circular path, holding it with the force 2.80 N.
How long does one revolution of the stone take?

9. A child is swinging a little stone (m = 289.4 g) on a string in a horizontal circular path, holding it with a force of 0.00290 kN. The period is 0.04678000 min. The radius of this uniform circular motion (in mm) is to be calculated.

a) Place a dot above the digits which are significant. How many significant digits do the given values have? How many significant figures does your final answer require?
b) Calculate the radius (in millimeters).
c) Round your result to the correct number of significant figures. Write your result in the normalized scientific notation (with a power of ten).

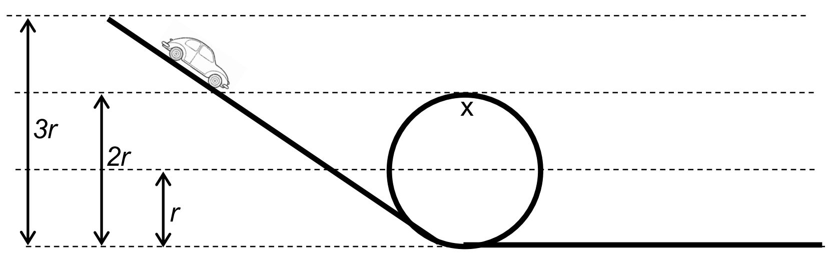
10. Ariel (m = 41.8 kg) is riding on a swing carousel. His chair is hanging from a rope of 5.50 m length.
Determine Ariel’s speed.



11. A roller coaster is to run a vertical looping of radius 5.2 m without falling down at the upmost point.
What is the minimum velocity required?

12. Cleopatra is swinging a bucket filled with water along a vertical circular path of radius 94.0 cm (see picture). The linear speed is constant at $3.40\ \frac{m}{s}$. At the highest point Cleopatra needs to hold the bucket with a force of 11.5 N.
What is the bucket's mass?

13. *Difficult, for ambitious folks only!*
A little car is travelling on a frictionless track with a looping of radius *r* (see picture below). Neglect air resistance.



What is the height *h* above ground from where the car needs to start so it does not fall down at the upmost point in the looping?
Find a mathematical expression that allows to calculate *h* from *r*.

Solutions:

1. a) $\pi = 180°$ b) $\frac{\pi}{2} = 90°$ c) $\frac{3\pi}{2} = 270°$ d) $\frac{\pi}{4} = 45°$ e) $\frac{\pi}{3} = 60°$ f) $\frac{2\pi}{3} = 120°$

2. magnitude, direction, center of the circle

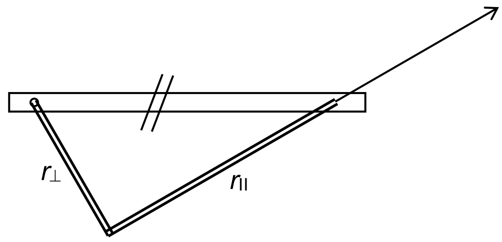
3. a) $F_G = m \cdot g = 97.55\ \text{kg} \cdot 9.81\ \frac{N}{kg} = 957\ N$

   the load is attached to three pieces of rope ⇒ $\frac{957\ N}{3} = \underline{\underline{319\ N}}$

   b) 319 N each (same amount of force everywhere in the rope)
   c) Add the forces exerted by two ropes pulling down and the weight of one pulley:
   $F = 2 \cdot 319\ N + 5.4\ N = \underline{643\ N}$ (holds for both A and B)
   d) If a third of the force is applied, three times as much rope must be pulled:
   $s = 3 \cdot 12.0\ m = 36.0\ m$

4. a) fourth hole from the right
   b) first hole from the right

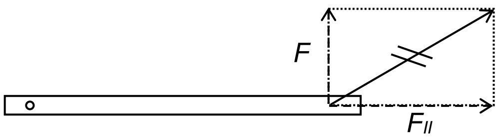
5. a)



$r_\perp = 2.0$ m
$r_{II} = 3.5$ m
$F = 2.5$ N

b) $M = F \cdot r_\perp = 2.5 \text{ N} \cdot 2.0 \text{ m} = \underline{\underline{5.0 \text{ Nm}}}$

c)



$F_\perp = 1.25$ N
$F_{II} = 2.2$ N
$r = 4.0$ m

d) $M = F_\perp \cdot r = 1.25 \text{ N} \cdot 4.0 \text{ m} = \underline{\underline{5.0 \text{ Nm}}}$

e) The results are the same :-)

6. Two pieces of rope are carrying the load with weight 2.0 N, that is 1.0 N each. At the point where the rope is attached to the lever, it pulls the lever upwards with a force of 1.0 N. The torque exerted on the lever (clockwise) is $M = F_{\text{rope}} \cdot r_{\text{rope}} = 1.0 \text{ N} \cdot 0.50 \text{ m} = 0.50$ Nm. The weight of the box exerts a counterclockwise torque. The force needed for equilibrium, applied by the box, is $F_{G(\text{box})} = \dfrac{M}{r_{\text{box}}} = \dfrac{0.50 \text{ Nm}}{0.25 \text{ m}} = \underline{\underline{2.0 \text{ N}}}$

7. a) $v = \dot{1}\dot{0}\dot{2}.\dot{9}\dot{7}\dot{5}\dot{0}\dot{0} \ \frac{\text{km}}{\text{h}}$: 8, $W_{\text{accelerating}} = 0.\dot{4}\dot{5}\dot{0}\dot{0}$ MJ: 4 result: 4

b) $m = \dfrac{2 \cdot W_{\text{accelerating}}}{v^2} = \dfrac{2 \cdot 450'000 \text{ J}}{\left(\frac{102.97500}{3.6} \cdot \frac{\text{m}}{\text{s}}\right)^2} = 1099.977773$ kg

c) $\underline{\underline{1.100 \cdot 10^3 \text{ kg}}}$

8. $F_C = m \cdot \omega^2 \cdot r = m \cdot \left(\dfrac{2\pi}{T}\right)^2 \cdot r$

$$T = \sqrt{\frac{m \cdot 4 \pi^2 \cdot r}{F_c}} = \sqrt{\frac{0.233 \text{ kg} \cdot 4 \cdot \pi^2 \cdot 1.79 \text{ m}}{2.80 \text{ N}}} = \underline{\underline{2.42 \text{ s}}}$$

9. a) $m = \dot{2}\dot{8}\dot{9}.\dot{4}$ g: 4 significant figures, $T = 0.0\dot{4}\dot{6}\dot{7}\dot{8}\dot{0}\dot{0}\dot{0}$ min: 7 significant figures, F_c: $= 0.00\dot{2}\dot{9}\dot{0}$ kN: 3 significant figures, result: 3 significant figures

b) $F_c = m \cdot \omega^2 \cdot r = m \cdot \left(\dfrac{2\pi}{T}\right)^2 \cdot r$

$$r = \frac{F_Z \cdot T^2}{m \cdot 4 \cdot \pi^2} = \frac{2.90 \text{ N} \cdot (0.04678000 \cdot 60 \text{ s})^2}{0.2894 \text{ kg} \cdot 4 \cdot \pi^2} = 1.99969 \text{ m} = 1'999.69 \text{ mm}$$

c) $\underline{\underline{2.00 \cdot 10^3 \text{ mm}}}$

10. $F_G = m \cdot g = 41.8 \text{ kg} \cdot 9.81 \ \frac{\text{m}}{\text{s}^2} = 410$ N

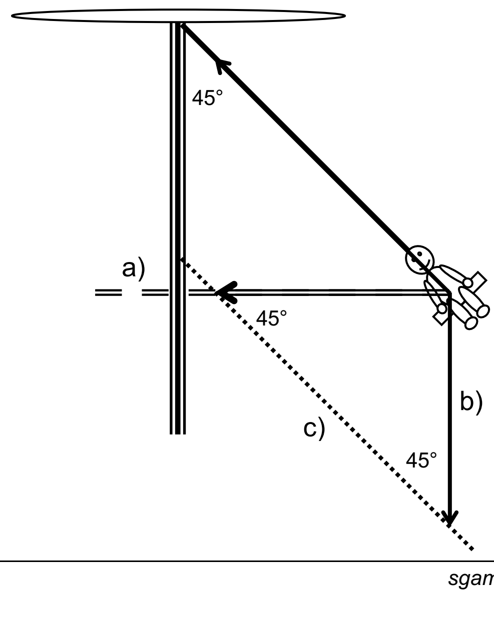
angle = 45°: Triangle of equal sides

$r = \dfrac{\ell}{\sqrt{2}} = \dfrac{5.50 \text{ m}}{\sqrt{2}} = 3.889$ m

$F_c = F_G = 410$ N

$F_c = \dfrac{m \cdot v^2}{r}$

$$v = \sqrt{\frac{F_Z \cdot r}{m}} = \sqrt{\frac{410 \text{ N} \cdot 3.889 \text{ m}}{41.8 \text{ kg}}} = \underline{\underline{6.2 \ \frac{\text{m}}{\text{s}}}}$$



11. For the roller coaster not to fall down, the condition $\frac{v^2}{r} \geq g$ needs to be fulfilled. Thus

$$v > \sqrt{g \cdot r} = \sqrt{9.81\ \frac{\text{m}}{\text{s}^2} \cdot 5.2\ \text{m}} = \underline{\underline{7.14\ \frac{\text{m}}{\text{s}}}} = \underline{\underline{26\ \frac{\text{km}}{\text{h}}}}$$

12. In the upmost point the direction of F_c is the same as the direction of F_G. Cleopatras hand needs to supply the force $F_{hand} = F_c - F_G$:

$$F_{Hand} = F_Z - F_G = \frac{m \cdot v^2}{r} - m \cdot g = m \cdot \left(\frac{v^2}{r} - g \right) \qquad m = \frac{F_{Hand}}{\frac{v^2}{r} - g} = \frac{11.5\ \text{N}}{\frac{\left(3.40\ \frac{\text{m}}{\text{s}}\right)^2}{0.940\ \text{m}} - 9.81\ \frac{\text{m}}{\text{s}^2}} = \underline{\underline{4.62\ \text{kg}}}$$

13. Condition for the car not to fall down at x: $\frac{v^2}{r} \geq g \qquad v \geq \sqrt{g \cdot r}$

$E_{p(gravitational)} = E_{kin} \qquad m \cdot g \cdot h = \frac{1}{2} \cdot m \cdot v^2$

$h = \frac{v^2}{2g} = \frac{g \cdot r}{2g} = \frac{r}{2}$ above the highest point of the looping x, that is $h = \underline{\underline{\frac{5r}{2}}}$ above ground.