Universal Law of Gravity

1. a) greater b) smaller

2. a) three b) one fourth c) one ninth d) sixteen

3. $m = \sqrt{\dfrac{F_G \cdot r^2}{G}} = \sqrt{\dfrac{2.5 \cdot 10^{-6}\ \text{N} \cdot (0.0125\ \text{m})^2}{6.67 \cdot 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}}} = \underline{\underline{2.42\ \text{kg}}}$

4. $m_{\text{object}} \cdot g_{\text{moon}} = G \cdot \dfrac{m_{\text{object}} \cdot m_{\text{moon}}}{r_{\text{moon}}^2}$ $\qquad g_{\text{moon}} = G \cdot \dfrac{m_{\text{moon}}}{r_{\text{moon}}^2}$

$$m_{\text{moon}} = \frac{g_{\text{moon}} \cdot r_{\text{moon}}^2}{G} = \frac{1.62\ \frac{\text{m}}{\text{s}^2} \cdot \left(1.737 \cdot 10^6\ \text{m}\right)^2}{6.67 \cdot 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}} = \underline{\underline{7.32 \cdot 10^{22}\ \text{kg}}}$$

5. $g_{\text{mercury}} = G \cdot \dfrac{m_{\text{mercury}}}{r_{\text{mercury}}^2} = 6.67 \cdot 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot \dfrac{3.29 \cdot 10^{23}\ \text{kg}}{\left(2.44 \cdot 10^6\ \text{m}\right)^2} = 3.7\ \dfrac{\text{m}}{\text{s}^2}$

6. a) $g\ (\text{at } 6.4\ \text{km}) = G \cdot \dfrac{m_{\text{earth}}}{(r_{\text{earth}} + s)^2} = 6.67 \cdot 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot \dfrac{5.97 \cdot 10^{24}\ \text{kg}}{\left(6'371 \cdot 10^3\ \text{m} + 6.4 \cdot 10^3\ \text{m}\right)^2}$

$= 9.79\ \dfrac{\text{m}}{\text{s}^2}$

b) $g\ (\text{at } 6'400\ \text{km}) = G \cdot \dfrac{m_{\text{earth}}}{(r_{\text{earth}} + s)^2} = 6.67 \cdot 10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot \dfrac{5.97 \cdot 10^{24}\ \text{kg}}{\left(6'371 \cdot 10^3\ \text{m} + 6'400 \cdot 10^3\ \text{m}\right)^2}$

$= 2.44\ \dfrac{\text{m}}{\text{s}^2}$

7. $g_1 = G \cdot \dfrac{m_1}{r_1^2}$ $\qquad m_2 = 2 \cdot m_1 \qquad r_2 = 3 \cdot r_1$

$$g_2 = G \cdot \frac{m_2}{r_2^2} = G \cdot \frac{2 \cdot m_1}{(3 \cdot r_1)^2} = \frac{2}{9} \cdot G \cdot \frac{m_1}{r_1^2} = \frac{2}{9} \cdot g_1 = \underline{\underline{0.22}}$$

8. $m_{planet} \cdot \omega^2 \cdot r_{planet\text{-}sun} = m_{planet} \cdot \left(\frac{2\pi}{T}\right)^2 \cdot r_{planet\text{-}sun} = G \cdot \frac{m_{planet} \cdot m_{sun}}{r^2_{planet\text{-}sun}}$

$$m_{sun} = \frac{(2\pi)^2 \cdot r^3_{planet\text{-}sun}}{G \cdot T^2} = \frac{(2\pi)^2 \cdot \left(5.79 \cdot 10^{10}\text{m}\right)^3}{6.67 \cdot 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot (87.97 \cdot 24 \cdot 3'600\ \text{s})^2} = \underline{1.99 \cdot 10^{30}\ \text{kg}}$$

9. a) Satellites always orbit around the center of the Earth. A geosynchronous satellite needs to move in the same way as the earth's surface does; therefore it has to orbit around the same axis as the earth revolves.
This is only possible if the satellite orbits within the equatorial plane.

b) 24 h

c) $m_{earth} = \frac{(2\pi)^2 \cdot r^3_{satellite\text{-}earth's\ center}}{G \cdot T^2_{satellite}} \quad \Rightarrow \quad r^3_{satellite\text{-}earth's\ center} = \frac{G \cdot m_{earth} \cdot T^2_{satellite}}{(2\pi)^2}$

$$r_{satellite\text{-}earth's\ center} = \sqrt[3]{\frac{G \cdot m_{earth} \cdot T^2_{satellite}}{(2\pi)^2}} =$$

$$= \sqrt[3]{\frac{6.67 \cdot 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \cdot 5.98 \cdot 10^{24}\ \text{kg} \cdot (86'400\ \text{s})^2}{(2\pi)^2}} = \underline{42'000\ \text{km}}$$

$$r_{satellite\text{-}earth's\ center} - r_{earth} = 42'000\ \text{km} - 6'370\ \text{km} \approx \underline{36'000\ \text{km}}$$