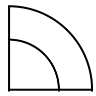
Circular motion

1. a) $360\,° = \underline{\underline{2\pi}} = \underline{\underline{6.28}}$ $\quad 90\,° = \underline{\underline{\frac{\pi}{2}}} = \underline{\underline{1.57}}$ $\quad 30\,° = \underline{\underline{\frac{\pi}{6}}} = \underline{\underline{0.52}}$ $\quad 5.93\,° = \underline{\underline{0.103}}$

   b) $2\pi = \underline{\underline{360\,°}}$ $\quad \frac{\pi}{4} = \underline{\underline{45\,°}}$ $\quad \frac{3\pi}{5} = \underline{\underline{108\,°}}$ $\quad 5.93 = \underline{\underline{340\,°}}$

2. A point farther from the center travels a longer way in the same time (see picture). Therefore the lineas speed is greater.



3. a) One minute = $\underline{\underline{60\text{ s}}}$

   b) $s = 2 \cdot \pi \cdot r = 2 \cdot \pi \cdot 2.2\text{ m} = \underline{\underline{14\text{ m}}}$

   c) $v = \frac{\Delta s}{\Delta t} = \frac{2\pi\, r}{T} = \frac{2\pi\ 2.2\text{ m}}{60\text{ s}} = \underline{\underline{0.23\ \frac{\text{m}}{\text{s}}}}$

   d) One quarter of a revolution: $\underline{\underline{\frac{\pi}{2}}} = \underline{\underline{90\,°}}$

   e) $\omega = \frac{\Delta\varphi}{\Delta t} = \frac{2\pi}{T} = \frac{2\pi}{60\text{ s}} = \underline{\underline{0.105\text{ s}^{-1}}}$

4. a) The angular speed is the same in both wheels. They roll off the road, and the linear speed of the wheels is the same as the speed of the vehicle.

   b) The smaller wheel turns faster. $v = \omega \cdot r$: If r is smaller, then ω must be greater (for equal v)

   c) The larger wheel's period is greater, because it turns slower: $T = \frac{2\pi}{\omega}$: The larger wheel's ω is smaller and therefore T is greater.

   d) The smaller wheel's frequency is greater. $f = \frac{\omega}{2\pi}$: If ω is greater, then f must be greater too.

5. a) $v = 100.0\ \frac{\text{km}}{\text{h}} = v = 100.0\ \frac{\text{km}}{\text{h}} = \frac{100.0}{3.6}\ \frac{\text{m}}{\text{s}} = \underline{\underline{27.8\ \frac{\text{m}}{\text{s}}}}$

b) $\omega = \frac{v}{r} = \frac{27.8\ \frac{\text{m}}{\text{s}}}{0.28\ \text{m}} = \underline{\underline{99.2\ \text{s}^{-1}}}$

c) $f = \frac{\omega}{2\pi} = \frac{99.2\ \text{s}^{-1}}{2\pi} = \underline{\underline{15.8\ \text{Hz}}}$

d) $T = \frac{1}{f} = \frac{1}{15.8\ \text{Hz}} = 0.0633\ \text{s}$ $\quad t = 10 \cdot T = 10 \cdot 0.0633\ \text{s} = \underline{\underline{0.633\ \text{s}}}$

e) $v = \omega \cdot r = 99.2\ \text{s}^{-1} \cdot 0.15\ \text{m} = \underline{\underline{14.9\ \frac{\text{m}}{\text{s}}}}$

6. $v = \frac{2\pi r}{T} = \frac{2\pi \cdot 1.5 \cdot 10^{11}\text{m}}{365 \cdot 24 \cdot 60 \cdot 60\ \text{s}} = \underline{\underline{29'886\ \frac{\text{m}}{\text{s}}}} = \underline{\underline{108 \cdot 10^{3}\ \frac{\text{km}}{\text{h}}}}$

7. a) $\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \cdot 60 \cdot 60\ \text{s}} = \underline{\underline{7.3 \cdot 10^{-5}\ \text{s}^{-1}}}$

b) $v = \omega \cdot r = 7.3 \cdot 10^{-5}\ \text{s}^{-1} \cdot 6'378 \cdot 10^{3}\ \text{m} = \underline{\underline{463\ \frac{\text{m}}{\text{s}}}} = \underline{\underline{1'670\ \frac{\text{km}}{\text{h}}}}$

8. $r = \frac{v \cdot T}{2\pi} = \frac{1.5\ \frac{\text{mm}}{\text{s}} \cdot 3'600\text{s}}{2\pi} = \underline{\underline{859\ \text{mm}}} = \underline{\underline{86\ \text{cm}}}$

9. a) In order to keep an object of twice the mass moving along a circular path (without changing the radius and angular speed), **twice** the centripetal force is required.

b) In order to keep an object moving along a circular path at twice the angular speed (without changing the radius and mass), **four times** the centripetal force is required.

c) In order to keep an object moving along a circular path at twice the distance from the center (without changing the angular speed and mass), **twice** the centripetal force is required.

10. a) $F_Z = m \cdot \omega^2 \cdot r = m \cdot \left(\frac{2\pi}{T}\right)^2 \cdot r = 75\text{ kg} \cdot \left(\frac{2\pi}{24 \cdot 60 \cdot 60\text{ s}}\right)^2 \cdot 6'378'000\text{ m} = \underline{\underline{2.5\text{ N}}}$

b) The gravitational force (attractive force between masses)

11. $F_Z = m \cdot (2\pi \cdot f)^2 \cdot r = m \cdot (2\pi)^2 \cdot f^2 \cdot r$

$$f = \sqrt{\frac{F_Z}{m \cdot 4\pi^2 \cdot r}} = \sqrt{\frac{100.0\text{ N}}{0.200\text{ kg} \cdot 4\pi^2 \cdot 0.500\text{ m}}} = \underline{\underline{5.03\text{ Hz}}}$$

12. a) The condition for the water to stay in the bucket is $\frac{v^2}{r} = a_Z > g$

$$\frac{v^2}{r} = \frac{\left(3.5\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\text{ m}} = 10.2\ \frac{\text{m}}{\text{s}^2} > 9.81\ \frac{\text{m}}{\text{s}^2} = g \qquad \underline{\text{yes!}}$$

b) The centripetal force needed to keep the bucket on a circular path is

$$F_Z = \frac{m \cdot v^2}{r} = \frac{4.5\text{ kg} \cdot \left(3.5\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\text{ m}} = 45.9\text{ N}$$

The weight of the bucket is $F_G = m \cdot g = 4.5\text{ kg} \cdot 9.81\ \frac{\text{N}}{\text{kg}} = 44.1\text{ N}$.

At the highest point the resultant force (centripetal force) is pointing downwards, towards the center of the circular path. The weight of the bucket is acting downwards, providing part of the centripetal force. The remaining part of the force is provided by the force of the hand pulling the bucket down: $F_{\text{centripetal}} = F_G + F_{\text{hand}}$
Therefore: $F_{\text{hand}} = F_{\text{centripetal}} - F_G = 45.9\text{ N} - 44.1\text{ N} = \underline{1.8\text{ N}}$

At the lowest point the resultant force (centripetal force) is pointing upwards towards the center of the circular path. The weight of the bucket is pulling it down, while the hand needs exert an upward force, holding it against the gravitational force, in addition to providing the required centripetal force: $F_{\text{centripetal}} = F_{\text{hand}} - F_G$ (the weight F_G and the centripetal force $F_{\text{centripetal}}$ are in opposite directions)
Therefore: $F_{\text{hand}} = F_{\text{centripetal}} + F_G = 45.9\text{ N} + 44.1\text{ N} = \underline{90\text{ N}}$

or:

top: $F_{\text{Schnur}} = F_Z - F_G = \frac{m \cdot v^2}{r} - m \cdot g = m \cdot \left(\frac{v^2}{r} - g\right) = 4.5\text{ kg} \cdot \left(\frac{\left(3.5\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\text{ m}} - 9.81\ \frac{\text{m}}{\text{s}^2}\right) = \underline{\underline{1.8\text{ N}}}$

bottom: $F_{\text{Schnur}} = F_Z + F_G = \frac{m \cdot v^2}{r} + m \cdot g = m \cdot \left(\frac{v^2}{r} + g\right) = 4.5\text{ kg} \cdot \left(\frac{\left(3.5\ \frac{\text{m}}{\text{s}}\right)^2}{1.2\text{ m}} + 9.81\ \frac{\text{m}}{\text{s}^2}\right) = \underline{\underline{90\text{ N}}}$