NEWTON'S DEMONSTRATION THAT PLANETS MOVE IN ELLIPSES

Hypoth. 1. Bodies move uniformly in straight lines unless so far as they are retarded by the resistence of y^e Medium or disturbed by some other force.
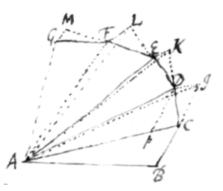
Hyp. 2. The alteration of motion is ever proportional to y^e force by wch it is altered.

Hyp. 3. Two Motions imprest in \two/ different lines, if those lines be taken in proportion to the motions & completed into a parallelogram, compose a motion whereby the diagonal of y^e Parallelogram shall be described in the same time in wch y^e sides thereof would have been described by those compounding motions apart. The motions AB & AC compound the motion AD.

Prop. 1.

If a body move in vacuo & be continually attracted toward an immoveable center, it shall constantly move in one & the same plane, & in that plane describe equal areas in equall times.

Let A be ye center towards wch ye body is attracted, & suppose ye attraction acts not continually but by discontinued impressions made at equal intervals of time wch intervals we will consider as physical moments. Let BC be ye right line in wch it begins to move from B & {illeg} wch it describes wth uniform motion in the first physical moment before ye attraction make its first impression upon it. At C let it be attracted towards ye center A w^{th}|by| one impuls or impression of force, & let CD be ye line in wch it shall move after that impuls. Produce BC to I so that CI be equall to BC & draw ID parallel to CA & the point D in wch it cuts CD shall be ye place of ye body at the end of ye second moment. And because the bases BC CI of the triangles ABC, ACI are equal those two triangles shall be equal. Also because the triangles ACI, ACD stand upon the same base AC & between two parallels they shall be equall. And therefore the triangles ACD described in the second moment shall be equal to ye triangle ABC described in the first moment. And by the same reason if the body at ye end of the 2^d, 3^d, 4th, 5^t & following moments be attracted by single impulses in <1r> D, E, F, G &c describing the line DE in ye 3^d moment, EF in the 4th, FG in ye 5^t &c: the triangle AED shall be equall to the triangle ADC & all the following triangles AFE, AGF & to the preceding ones & to one another. And by consequence the areas compounded of these equall triangles (as ABE, AEG, ABG &c) are to one another as the {l}ines times in wch they are described. Suppose now that the moments of time be diminished in length & encreased in number in infinitum, so y^t the impulses or impressions of ye attraction may become continuall & that ye line BCDEFG by ye infinite number & infinite littleness of its sides BC, CD, DE &c may become a curve one: & the body by the continual attraction shall describe areas of this Curve ABE, AEG, ABG & proportionall to the times in wch they are described. W. W. to be Dem.